PLANCK SCALE PHYSICS , PREGEOMETRY AND THE NOTION OF TIME

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1. Introduction

Recent progress in quantum gravity [1] and string theory [2] has raised interest among scientists to whether or not nature behaves "discretely" at the Planck scale. However, it is not clear what this metaphor means or how it should be implemented into systematic study concerning physics and mathematics in the Planck regime.

There are basically two attitudes towards this discreteness at Planck scale. One starts from continuum concept and then tries to detect or create modes of "discrete behaviour" on finer scales. We call this as "top down approach" [3]. On the other hand, one can try to describe how macroscopic space-time or its underlying mesoscopic substratum emerges from a more fundamental concept like fluctuating cellular network [4] around the Planck scale. We call this as bottom up [5] approach. It is generally believed that no physical laws which are valid in continuum space-time will be valid beyond or around the Planck scale. The very concept of space-time, causality may not be valid beyond Planck scale. Some scientists suspect space or time should be considered as emergent properties [6].

In a recent paper [5] we developed a kind of pre geometry around the Planck scale where notion of "clock time" is proposed below or near the Planck scale. This is not to be confused with the physical time. It is possible to understand this time if one defines a category of generalized time [7] using the idea of sheaf cohomology. This generalized time may not be necessarily linear in nature. There is no before or after in this category. Physical time with its usual characteristics emerges from this generalized time under appropriate conditions. . Then one gets linearity and ordering (like before and after) in the sequence of time. This ordering of the sequence can be thought as an emergent property.

In this paper, we shall briefly describe the developments of quantum gravity and string theory so as to understand the conceptual problems related to the discrete behaviours at the Planck scale in section 2. In section 3, we shall describe the pre geometry, cellular networks and overlapping fuzzy lumps. Then we shall introduce the concept of generalized time and the emergence of physical time in section 4. Finally, a few remarks concerning the possible physical and philosophical implications have been made in section 5.

2. New Challenges in 20th Century Physics

The birth of quantum mechanics in early twentieth century shattered our idea whether it is possible to describe our physical world with one physical law. The world on large scale - the motion of terrestrial objects like planetary motion, stars and galaxies down to our day to day world are explained with the help of Newtonian mechanics. But as we move to the smaller scales, regarding the behaviour of objects with small masses like electrons, protons and photons, one needs to consider different physical theory as quantum theory. So we have two different types of physical law that are valid at two different levels of the physical world. However, the popular belief among the physicists is that if we really understand quantum theory, it is possible to use it to describe the large-scale phenomena i.e. what we call classical world. In practice, however, we use either classical physics or quantum physics. Penrose [8] curiously noticed " how the ancient Greeks looked at the world... one set of laws ... applied to earth and different set.. in the heavens."

Let us now look at the scales which we deal with the physical world. The time and length scales at the bottom are known as Planck time and the Planck length respectively. Planck time is 10^{-43} sec which is considered the shortest time in our physical world i.e. shorter than the shortest lived particles, called resonance, which is about 10^{-23} sec. Likewise, the Planck length which is about 10^{-33}cm, is considered the shortest length i.e. fundamental unit of length. Now, when one needs to combine both Planck time and Planck length, it is necessary to consider both quantum theory and general relativity. Quantum theory is valid for small length scales and general relativity for large length, and time, scales. As soon as both. quantum theory and general relativity are brought together, one needs to consider both Planck length and Planck time. For example, if we need to describe the physics of black holes or the universe at the the big bang, it is necessary to consider both quantum theory and general relativity. But, the attempt to combine the quantum formalism with general relativity leads to catastrophe rather than the harmony in nature.

Twentieth-century researche in quantum gravity [1] changed our focus into the physical world. Before merging of quantum theory and general relativity, one needs to consider as the microscopic nature of space-time i.e. as we go to the smaller and smaller length scales, the nature of space-time should be studied with uncertainty principle . From the point of view of classical mechanics, the space remains flat at arbitrary small lengths. Quantum mechanics changes this conclusion. Everything in this world even the gravitational field is subject to quantum fluctuations inherent in the uncertainty principle. The uncertainty principle indicates that the size of the fluctuation of gravitational field becomes larger as we focus our attention to the smaller regions of space. These quantum fluctuations might be manifested as violent distortions of the surrounding space and hence the curvature related to the gravitational field. John Wheeler [9] described it as quantum foam in which the notions of left and right, before and after lose their meaning. At this scale, we face the fundamental incompatibility between general relativity and quantum theory. Essentially, the notion of smooth geometry that is prerequisite for general relativity is not a valid concept at small scales due to violent fluctuations of the quantum world at. Now if we consider the distances used in everyday life, the random or violent fluctuations concel each other and the concept of smooth geometry is valid. .This is similar to a picture composed of dots. When viewed from afar, the picture presents a smooth image. However, if we view it from a very small distance or at finer scales, we see nothing but a collection of dots, each separate from the other. This pictire's discrete structure appears upon examines at small scales. The fabric of space-time looks similar i.e. discrete at the smallest scale (Planck length) and smooth at large scales.

The various contradictory attempts to incorporate gravity into a quantum mechanical framework leads to new a search for deeper understanding of the nature. Green et al [10] presented a convincing evidence that superstring theory might provide this understanding. Strings are considered as one-dimensional filament like objects, vibrating to and fro. Accordingly, the elementary ingredients of the universe are not now point like objects but one-dimensional filaments. String theory proposes that strings are ultramicroscopic ingredients making up the particles from which atoms themselves are made. On average, strings are of the size of Planck length. Proposing strings as fundamental ingredients of the universe has far reaching consequences. In particular, String theory appears to resolve the conflict between general relativity and quantum mechanics.

Despite its elegance, string theory has not so far led to any new predictions concerning the properties of elementary particles. Moreover, for strictly technical reasons string theory requires nine or even ten spatial dimensions. The emergence of these extra spatial dimensions is probably one of the most difficult issues, unless one appeals solely to mathematical formalism. In fact, it is very difficult to get an intuitive non technical reason for the existence of these extra dimensions. Rutherford once commented that if you can't explain the result in simple non technical term, you really don't understand it. Strings are considered as fabric of space-time like a piece of material out of which the universe is tailored.. This may give rise to new possibility to understand the space-time at small scales. But what do we really mean by the fabric of the Universe? This question has been debated for hundreds of years. We can summarize it briefly as follows :

- Newton declared space and time as the eternal and immutable ingredients in the make up of the cosmos.

- Leibniz [11] dissented, claiming that space and time are merely bookkeeping devices for summarizing relationship between objects and events in the universe. The location of an object in space and in time has meaning only in comparison with another object.

- Mach [12] further developed Leibniz's view, which is much closer to the view of today's physicists.

After the proposal of string theory, the imminent question arises whether or not we should view ourselves as truly being embedded in something when we refer to our immersion in the space-time fabric. However, the string theory does not suggest an answer to this question. Moreover, an ordinary piece of fabric is the end product of raw material being carefully woven. In the raw state, before strings, there is no notion like "before". Maybe, our language is not so developed to handle it. Is this world really comprehensible?

3. Planck Scale and Pregeometry

It is evident from the above analysis that physicists do not have a satisfactory model of the physical world at the Planck scale. In our search for such a model, our working philosophy is that the continuum concepts of ordinary physics and/or mathematics can be reconstructed from more primordial pre geometric (basically discrete) concepts , which are prevalent at the Planck scale. Here, geometry emerges from a purely relational picture *a' la* Leibniz. In particular, the underlying substratum of our physical world or, more specifically the space-time (quantum) vacuum can be viewed as cellular network [4]. This discrete structure consists of elementary nodes, n_i, which interact or exchange information with each other via bonds, b_{ik}, playing the role of irreducible elementary interactions. The possible internal structures of the nodes, modules or bonds (interaction channels) are described by a discrete internal state space carried by the nodes/bonds. The node set is assumed to be large but finite or countable. The bond b_{ik} is assumed to connect the nodes n_i, n_k. The internal state of the nodes/bonds are denoted by s_i, J_{ik} respectively. Our philosophy is to generate complex behaviours out of simple models.

Let us choose

$$s_i \in q.\mathbf{Z}, \quad J_{ik} \in -1, 0, +1$$

with q an elementary quantum of information. In our approach, the bond states are dynamical degrees of freedom which, *a fortiori*, can be switched off or on. The wiring, that is the pure geometry of the network is also an emergent, dynamical property and is not given in advance. Consequently, the nodes and the bonds are not arranged in any regular way i.e. a lattice, and there is no fixed near/far order. This implies that geometry will become to some extent a relational (Machian) concept and is not *a priori* element. In this model, the nodes and bonds are updated in discrete clock time steps, $t = z.\tau, z \in \mathbf{Z}$ and τ being an elementary clock time interval. This updating is given by some local dynamical law. Here, local means that the node/bond states are changed at each clock time step according to a prescription with input to all states of some neighbourhood (in some topology) of the node/bond under discussion. Here, t is not to be confused with the physical time, which is also expected to be an emergent coarse grained quantity.

Now we shall introduce the concept of generalized time and its relation to physical time.

There are many different aspects of our class of cellular networks. Our cellular netorks can be regarded as complex dynamical systems, or statistical / stochastic frameworks, but in purely geometric sense, they are evolving graphs. It is possible to describe the evolution and structure of large dynamical graphs. Furthermore, the network at any clock time encodes the complete near and the far-order structure of the network at other clock times. It tells us the relative proximity of network subsets in terms of possible physical aspects such as strength of correlations or statistical distance. Stochastic aspects result from the underlying network law, which induces, among other things, a certain amount of creation and annihilation of bonds among the nodes. As a consequence the size and shape of the cliques or lump fluctuates in course of network evolution. This derived coarser network i.e. the clique graph or web of lumps, is defined by using meta-nodes to represent the cliques and meta-bonds to represent overlap of cliques.

It is to be noticed that while this new network may be regarded as being coarser in some sense, in general it may nevertheless consists of many more nodes and bonds than the underlying primordial network. Usually there are many more maximal subsimplices than primordial nodes, as a given node will typically belong to quite a few different subsimplices. This array of intersecting maximal subsimplices has the natural structure of a simplical complex with smaller simplices as faces of maximal ones i.e. the cliques. If we represent this simplical complex by a new (clique) graph with only the maximal simplices occurring as meta-nodes, we loose, on the other side, some information, as we do not keep track of situations where, say, three lumps or cliques have a common overlap. It is possible to make some kind of ensemble averages over fluctuating but individual cliques and identify them with fuzzy lumps. The underlying philosophy is that the space (set of points) is replaced by a class of functions on this space. This is similar to the philosophy of non-commutative geometry. The underlying graph carries a natural distance function

$$d(n_i, n_k)$$

i.e., the minimal length of a path connecting the given nodes where the length of a path is simply the number of bonds comprising it. Keeping the labeled nodes or cliques fixed, the distance fluctuates in clock time. The clique metric will fluctuate since the cliques change their shape and size i.e. also their degree of overlap. Now when we switch from the above kind of dynamical picture of a time dependent graph, the ensemble picture of fuzzy cliques or lumps, our point of view changes to a static but probabilistic one in the spirit of Menger [13]. Here, the structure of the space is no longer time dependent while its largely hidden dynamics is now encoded in various probabilistic notions.

4. Pregeometry and Notion of Time

In the above analysis we have discussed two approaches to the geometric structure at Planck scale. In one approach, we have considered underlying network with dynamical laws and the static picture of fuzzy cliques or lumps with probabilistic notion in other approach. It raises the age old dilemma regarding the physical world and the mathematical laws. There are people who prefer to think of mathematical concepts merely as idealization of our physical world. Here, the mathematical world is considered to be emerging from the world of physical objects. The other group prefers to think of the physical world as emerging out of the "timeless" world of mathematics. Penrose [8] remarked "one of the remarkable things about the behaviour of the world is how it seems to be grounded in mathematics to a quite extraordinary degree of accuracy...more we understand about the physical world, and the deeper we probe into the laws of nature, ... the physical world almost evaporates and we are left only with mathematics". Our working philosophy to understand the physical world around Planck scale is that at this stage of development as physical laws beyond Planck scale are yet to found, we shall start with mathematical concepts and mathematical laws and observe how the physical world emerges.

The recent developments in category theory and sheaf cohomology [7] shed new light at understanding the notion of time below Planck length. Category theory was created by Eilenberg and MacLane[14] in the forties. It provides a powerful and very general methods in algebraic geometry and algebraic analysis.

Let us start with some fundamental concepts in category theory. A category consists of objects and morphisms. For each pair of objects X and Y of $\mathbf{C}$, we have the set $\mathrm{Hom}_{\mathbf{C}}(X,Y)$ of morphisms from X to Y. For morphisms f from X to Y and g from Y to Z, the composition $g\ o\ f$ of g and f is defined and the composition $g\ o\ f$ is a morphism from X to Z, which satisfies the associate law :

$$h\ o(g\ of) = (h\ og)\ o\ f$$

For each object X, there is a morphism, called the identity morphism id_x, from X to X itself, satisfying $id_x\ o\ f = f$ and and $g\ o\ id_x = g$ for any morphism of g from X to X.

For example : (a) The category of sets consists of objects being sets and morphisms being set theoretic maps. (b) The category of abelian groups as its objects and group homomorphism as its morphism.

Functor is an important concept in category theory. A Functor $\mathbf{F}$ from a category $\mathbf{C}$ to category $\mathbf{C}'$ is defined as : for each object X in $\mathbf{C}$, $\mathbf{F}$ assigns an object $\mathbf{F}\ X$ in $\mathbf{C}'$, such that for each morphism

$$f : X \to Y \text{in } \mathbf{C}$$

$\mathbf{F}$ assigns a morphism

$$\mathbf{F}f : \mathbf{F}X \to \mathbf{F}Y \in \mathbf{C}'$$

Then $\mathbf{F}$ must satisfy

$$\mathbf{F}id_x = id_{\mathbf{F}X}$$

and

$$F(gof) = \mathbf{F}go\mathbf{F}f$$

A contravariant functor is a covariant functor from the dual category $\mathbf{C}^{\text{opp}}$ of $\mathbf{C}$ to $\mathbf{C}'$. Given a category $\mathbf{C}$, a new category $\mathbf{C}^{\text{opp}}$, called the dual category of $\mathbf{C}$, is obtained in the following manner.

(i) The objects of the category $\mathbf{C}^{\text{opp}}$ coincide with the objects of the category $\mathbf{C}$.

(ii) The set of morphisms $\text{Hom}_{\mathbf{C}^{\text{opp}}}(XY)$ is identical with $\text{Hom}_{\mathbf{C}^{\text{opp}}}(YX)$

(iii) The composition map $\text{Hom}\mathbf{C}^{\text{opp}}(XY)x\text{Hom}\mathbf{C}^{\text{opp}}(\mathbf{YZ}) \to \mathbf{C}^{\text{opp}}(\mathbf{X}, \mathbf{Z})$

So

$$(\mathbf{C})^{\text{opp}} = \mathbf{C}$$

The concept of dual category enables one to dualize each notion and each statement with respect to a category $\mathbf{C}$ into a notion and a statement with respect to the category $\mathbf{C}^{\text{opp}}$.

An important example is a presheaf. The concept of presheaf and sheaf play significant role in category theory. The sheaf (cohomology) has been employed as a bridge from local information to global information. Let $\mathbf{T}$ be a topological space i.e. Euclidean n-space $\mathbf{R^n}$. A presheaf F is an assignment; for every open subset V of $\mathbf{T}$,

$$V \to FV$$

where FV is an object in a category satisfying some contravariant functor axioms. Now if a presheaf is given, one can ask whether it is possible to obtain a global information from a collection of local data by "pasting" those local data. The answer is "yes" if the presheaf further satisfies some axiom. This means that if a presheaf F is actually a sheaf, then not only the discrete information data $\mathbf{F}(\mathbf{V_i})$ can be obtained for each covering, but also global information can be obtained by gluing the local data.

Let us now define $\hat{T}$ as a category of presheaves on the category $\mathbf{T}$ associated with a topological space T with values in a product category $\prod_{\alpha\in\Gamma} \mathbf{C}_\alpha$. More precisely, $\hat{T}$ is the category of contravariant functors from the category $\mathbf{T}$ associated with a topological space T to a product category $\prod_{\alpha\in\Gamma} \mathbf{C}_\alpha$ where Γ is an index set. The category $\mathbf{T}$ is said to be the generalized time space or generalized time category [7] when the real line $\mathbf{R}$ is embeddable in T. Namely,

$$\hat{T} = \prod_{\alpha \in \Gamma} \mathbf{C}_\alpha{}^{\mathbf{T}^{opp}}$$

.

To be more explicit, for an object V in T i.e. an open set V of T, and for an object P in $\hat{T}$, we have $P(V) = (P_\alpha(V))$,
$\alpha \in \Gamma$ where each $P_\alpha(V)$ is an object of $\mathbf{C}_\alpha$. Recall that by the definition an entity is a presheaf P in $\hat{T}$ where $\mathbf{C}_\alpha, \alpha \in \mathbf{\Gamma}$ represents the totality of physical categories. It should be noted that $\hat{T}$ includes matter like elementary particles, atoms, molecules etc. The index set Γ may be divided into several parts. The first part of Γ is used for physical world categories. We will use integers as indices for physical categories:

$$\mathbf{C_j}, \quad \mathbf{j = 0, 1, 2, ... \in \Gamma}$$

where $\mathbf{C_0}$ is the generalized time category T itself, $\mathbf{C_1}$ is the micro world and $\mathbf{C_2}$ is the macro world. $\mathbf{C_1}$&$\mathbf{C_2}$ are discrete categories with structures. Now, the time what we experience or use in physical world, is a linear and uni-dimensional space i.e. the real line R. This allows us to introduce the notion of before and after i.e. past , present and future. We make the following assumption:

There exists an embedding

$$\mathbf{i} : \mathbf{R} \to \mathbf{T}$$

. Here, the presheaf P restricted to the closed subset $\mathbf{i}(\mathbf{R})$ of $\mathbf{T}$ is a presheaf i^{-P} over $i(R)$. In this world, several worlds exist simultaneously with respect to generalized time.

The cohomology of network of entities can be constructed in the following manner. One defines a complex Σ in a category say $\mathbf{C}$ as a sequence of entities with morphisms in chain from one object to the other, the composition of two consecutive morphisms being a zero morphism. The sequence

$$\xrightarrow{\gamma} P(U) \xrightarrow{\delta} Q(U) \xrightarrow{\phi} R(U) \to .....$$

is such that this forms a cochain complex, namely, any consecutive composition of morphisms in the above sequence is trivial. The cohomology at $Q(U')$ denoted by $H^\star(.... \to Q(U) \to ...)$ is defined as the subquotient.

Now, if there is one entity Q, the above sequence becomes,

$$.... \to 0 \to Q(U) \to 0 \to .....$$

. Then the cohomology at $Q(U)$ is $Q(U)$ itself. That is the sub object of $Q(U)$ which has no influence on anyone is the whole $Q(U)$. Consider the above sequence for two entities as

$$...... \to 0 \to P(U) \xrightarrow{\delta} Q(U) \to 0 \to ...$$

. Then the cohomology at $Q(U)$ is the quotient $\frac{Q(U)}{Im\delta_{ij}}$ i.e., the cohomology at $Q(U)$ is the quotient obtained by regarding the influence or information $Q(U)$ receives from $P(U)$ as the trivial part of $Q(U)$. On the other hand, the cohomology at $P(U)$ is the subobject $\mathrm{Ker}\delta_{ij}$. Ker denotes the kernel and Im be the Imaginary part. In this manner one can construct the cohomology for sequence of many entities and the influence of influence will not be lost.

5. Implications

The above analysis shows that it is possible to construct a pre geometric framework so as to describe the physics around Planck scale. At the deepest level i.e. beyond Planck level, we proposed purely mathematical concepts like category of generalized time. As soon as we impose certain restriction or condition, we get physical time around or above Planck scale. This generalized time category is called "noumenon" of infinite time and imposing restriction we get "phenomenon" time or physical time. This conditioning is intimately related to defining measures and also to perception. By using different ways of conditioning we can have the notion of time at different scales or level of the physical world namely in classical mechanics, theory of relativity or in quantum domain. So the physical time seems to be emergent property due to conditioning or limitations.

There is a great deal of debates specially at philosophical level about the concept of emergence. Butterfield and Isham[6] made an extensive review on this topic. In everyday language, emergence is considered to be a process in temporal sense. But in our framework at the level beyond Planck scale. In generalized time category there is no concept of "before" or "after". So "emergence" should be considered in non-temporal sense. Here, "emergence" can be thought of associated with "conditioning" or "imposing limitation in measurement". The reality beyond Planck scale should be considered as "Veiled reality" of d'Espagnat [15]. He suggested that it is necessary to speak of an independent reality which can't be described in the sense of traditional realism. One can get statistical knowledge of it.

In Perennial Indian Philosophy [16] this kind of "veiling" is described by the term "Maya" or "Illusion". In Sanskrit language the generic meaning of the word "Maya" is related to "Measure" and hence the limitation. Again the word "emergence" has also been described in a completely different way rather than western sense. It is said that "emergence" is related to the concept "expansion from within without". This is not an expansion from a small center or focus but without reference to size or limitation or area, means the development of "limitless subjectivity into as limitless objectivity". Accordingly, this expansion not being an increase in size -for infinite extension (generalized time) admits no enlargement - is just a change of condition i.e. from pre geometry to discrete Planck scale and then to continuum space-time. Here, the expansion is traced back to its origin to a kind of primordial (and periodic) vibration which is responsible for the manifestation of the physical objects and physical world.

It needs further elaboration to find the linkage between the fluctuations associated with the creation or annihilation of nodes or creation or annihilation of black holes as discussed in our above cellular network model. This fluctuation might be responsible for the fuzzy character of the lumps or foamy space -time as speculated by Wheeler [9].

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