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Causal analysis of asymmetric entangled states under decoherence

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ABSTRACT

An application of quantum causal analysis to asymmetric entangled states under decoherence is considered. Two models of decoherence: dissipation and depolarization are studied. For the both models the strength and the direction of induced causality has been computed. It turns out that the decoherence acting along original causality destroys entanglement to a lesser degree than it acting against this causality.

1. Introduction

The decoherence is a key problem which impedes quantum computers creation. Therefore the exploration of an entanglement decay processes is very important in quantum information physics. Ref. [1] poses the problem of a dependence between bipartite state asymmetry and a rate of entanglement decay, when only one subsystem interacts with an environment. The study considers bipartite entangled state ρ_{AB} which satisfies an inequality

$$S_B < S_{AB} < S_A, \tag{1}$$

where S_A, S_B and S_{AB} are von Neumann entropies of two subsystems and the whole system respectively. In view of the fact that the inequality $S_{AB} < S_A$ can be realized only for quantum variables, the subsystem A was called "quantum"; contrastingly, subsystem B was called "classical". A model of discrete dynamics of entanglement, which involves global unitary dynamics of the system and periodic actions of depolarizing channel, of such asymmetrical system was considered. As a result, it was discovered that the entanglement decreases faster if the depolarizing channel acts on the "classical" subsystem. This counterintuitive effect was called subsequently an anomalous entanglement decay.

In the present Letter we consider an implementation of quantum causal analysis [2,3] to such "quantum–classical" states. This new method is intended for the asymmetric systems analysis and reveals interesting peculiarities of entanglement decay processes in the different models of decoherence.

The Letter is organized as follows. In Section 2 we briefly describe the principles of quantum causal analysis and shortly overview some previous results to gain greater insight into the subject of this Letter as part of a broader problem. In Section 3 we describe two different models of decoherence: dissipation and depolarization. In Section 4 we apply the quantum causal analysis to asymmetric "quantum-causal" state under dissipation and depolarization. Then we compare the behavior of causality in these processes with corresponding decay of entanglement. The results of the Letter are summed up in Section 5.

2. Quantum causal analysis

Causal analysis is a new method, which propose an original approach to causal connection understanding. Firstly it was designed for classical variables (e.g. [4]) and later it was extended and applied to quantum variables [2,3]. Causal analysis is based on the idea that there is an information asymmetry in the cause–effect relation. Indeed, in spite of the fact that principle of causality is widely used in physics, it usually means no more than retardation of the effect relative to the cause. However, the retardation is necessary but it is not a sufficient condition of the causal connection ("Post hoc non est propter hoc"). The fact that in simple situations we usually are able to realize what is a cause and what is an effect without retardation measurement indicates that these conceptions are asymmetrical (e.g. there is obvious casual relationship between the currents of a lamp and of a photocell circuits). The problem is to formally define this asymmetry without resort to time relation, which has to be introduced as an axiom after the definition is made. It is actual for complicated situations in systems with feedbacks, where usual intuitive understanding of causality may lead

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to misunderstandings. The key idea of causal analysis can be formulated as follows: in terms of information transmission process, the cause influences the effect to a greater extent than the effect influences the cause.

Mathematical formalization of causal analysis is founded on a pair of independence functions

$$i_{A|B} = S_{A|B}/S_A,$$
$$i_{B|A} = S_{B|A}/S_B, \tag{2}$$

where S_A, S_B, $S_{A|B}$, $S_{B|A}$ are marginal and conditional entropies for two variables A and B (for the classical variables Shannon entropies are used, for the quantum ones – von Neumann entropies). It is easy to see that for the classical variables $i \in [0, 1]$. The meaning of the independence functions is quite transparent: at $i_{B|A} = 1$ B is independent of A; at $i_{B|A} = 0$ B is one-valued function of A. In other words, the values $1 - i$ determine the unilateral dependences of the variables. $i_{A|B} \neq i_{B|A}$ means that the causal connection between subsystems takes place. It is also can be observed that for the quantum variables $i \in [-1, 1]$ (since the conditional entropy can be negative in entangled states). In particular, for the bipartite pure entangled states from Schmidt decomposition it is follows that $i_{A|B} = i_{B|A} = -1$, thus zero independence functions correspond to maximal classical correlations; negative independence functions correspond to quantum correlations. In any case the inequality $i_{A|B} > i_{B|A}$ qualitatively indicates that A is the cause and B is the effect.

As a quantitative measure of causality the linear velocity of irreversible information flow c_2, called the course of time, was derived [2,3] (the notation followed Kozyrev's pioneer work on causal mechanics [5], where originally although in less rigorous terms a pseudoscalar of the same meaning was introduced):

$$c_2 = k\frac{(1 - i_{A|B})(1 - i_{B|A})}{i_{A|B} - i_{B|A}} \tag{3}$$

where $k = \Delta r/\delta t$ is a dimensional coefficient which is defined by an effective distance Δr and brachistochrone evolution time δt [6]. Then the formal definition of causality connection is introduced: "*The cause A and the effect B are the subsystems for which* $c_2 > 0$". Negative values of c_2 mean that B is the cause and A is the effect. The absence of causal connection corresponds to $|c_2| \to \infty$. So the less $|c_2|$, the stronger causality is.

To keep the examples described bellow from becoming too sophisticated, we shall restrict ourselves to calculations of c_2 with $k = 1$, because, as it has been demonstrated in Ref. [2], the precise estimation of c_2 with regard to variable k, calculated through the eigenvalues of Hamiltonian, does not lead to a qualitative change in c_2 behavior.

Then we can formulate the axiom for retardation τ of the effect relative the cause in classical causality:

$$c_2 > 0 \quad \Rightarrow \quad \tau > 0,$$
$$c_2 < 0 \quad \Rightarrow \quad \tau < 0,$$
$$|c_2| \to \infty \quad \Rightarrow \quad \tau \to 0. \tag{4}$$

It should be pointed out that nonlocal correlations are often treated as instantaneous and causeless. Our approach includes such treatment, but only as a particular case. Ref. [2] shows that in all pure states $|c_2| \to \infty$ and only in the mixed states c_2 can be finite. Therefore the mixedness turns out to be a necessary condition for quantum causality.

In Ref. [7] Cramer was the first to distinguish the principles of strong and weak causality. The strong (local) causality corresponds to the usual condition for retardation of the effect relative to the cause described by (4). Without this axiom we have the weak causality, which corresponds only to nonlocal correlations and implies a possibility of information transmission in reverse time, but only related with unknown states (hence "the telegraph to the past" is impossible). Note that in the examples of Section 4 we shall nowhere use the axiom (4). Reverse time is allowed at least in a transactional sense [8]. Moreover (although it is not very important for the present Letter scope) it should be mentioned, that weak causality admits the extraction of information from the future without the well-known classical paradoxes. The experimental possibility of detection of such time reversal phenomenon was theoretically predicted by Elitzur and Dolev [9] and really proved for the intramolecular teleportation [10] and for the macroscopic entanglement, e.g. [11].

Relation between the measure of causality c_2, the different measures of entanglement, mixedness and concrete system and environment parameters has been studied in the wide range of examples of the two- [2] and three- [3] qubit states. Some results turned out to be rather nontrivial: for instance, one of particular results is that for thermal entanglement under a nonuniform external magnetic field the effect always corresponds to the region of stronger field, and although directionality of causal connection is unaffected by temperature, but its value is affected by temperature oppositely in the parallel and antiparallel fields. Another general result is that causality can be induced by asymmetric decoherence.

In the rest of the Letter we consider an implementation of the quantum causal analysis to the different variants of decoherence with quite interesting consequences.

3. Decoherence models

Let us consider two simplest models of decoherence: dissipation and depolarization. In line with Refs. [12,13] the dissipation process is described by the following transformation of corresponding density matrix elements:

$$|0\rangle\langle 0| \to |0\rangle\langle 0|,$$
$$|1\rangle\langle 1| \to (1 - p)|1\rangle\langle 1| + p|0\rangle\langle 0|,$$
$$|0\rangle\langle 1| \to \sqrt{1 - p}|0\rangle\langle 1|,$$
$$|1\rangle\langle 0| \to \sqrt{1 - p}|1\rangle\langle 0|, \tag{5}$$

where $p \in [0, 1]$ defines a decoherence degree. Thereby the complete dissipation turns a subsystem under decoherence into the state $|0\rangle\langle 0|$ with zero von Neumann entropy. In terms of causal analysis it means that corresponding subsystem turns into a sink of information or, in other words, into an effect.

The depolarization can be depicted by the following transformation of corresponding density matrix elements [12,13]:

$$|0\rangle\langle 0| \to (1 - p)|0\rangle\langle 0| + p\mathbf{I}/2,$$
$$|1\rangle\langle 1| \to (1 - p)|1\rangle\langle 1| + p\mathbf{I}/2,$$
$$|0\rangle\langle 1| \to (1 - p)|0\rangle\langle 1|,$$
$$|1\rangle\langle 0| \to (1 - p)|1\rangle\langle 0|, \tag{6}$$

where $\mathbf{I}$ is identity matrix. From this transformation we notice that the complete depolarization turns a subsystem under decoherence into the state $\mathbf{I}/2$ with maximal von Neumann entropy (1 bit). In terms of causal analysis it means that corresponding subsystem turns into a source of information or, in other words, into a cause.

Thus we have two fundamentally different versions of the decoherence process with opposite directions of induced causality.

4. Causality and entanglement

In line with Ref. [1] let us consider general form of asymmetric bipartite entangled state:

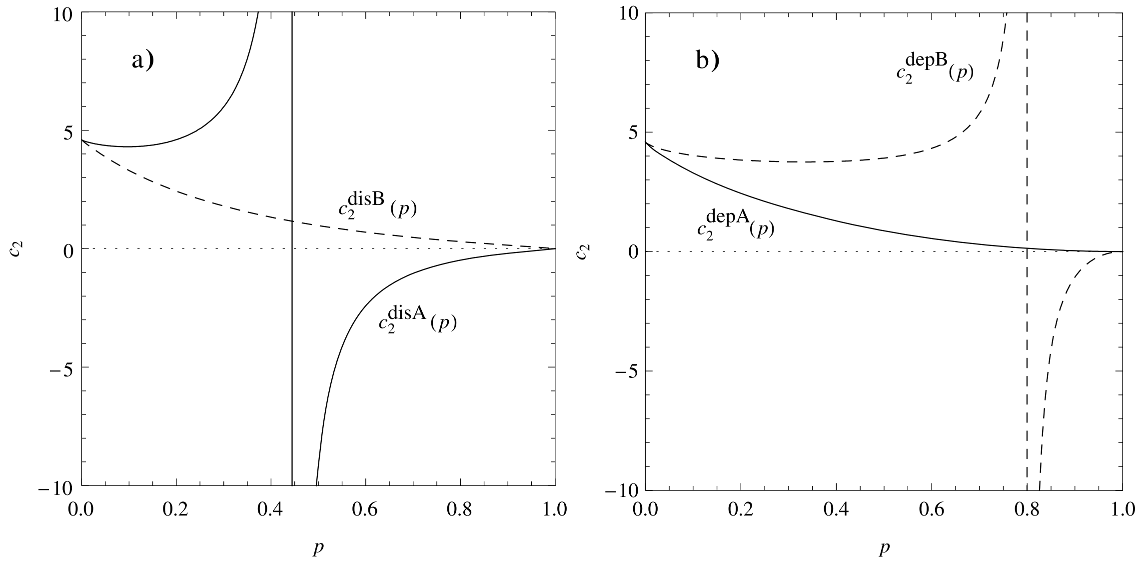

Fig. 1. Linear velocity of irreversible information flow c_2 as a measure of causality in the different cases of decoherence: (a) dissipation (5) of subsystem A (solid line) and subsystem B (dashed line); (b) depolarization (6) of subsystem A (solid line) and subsystem B (dashed line).

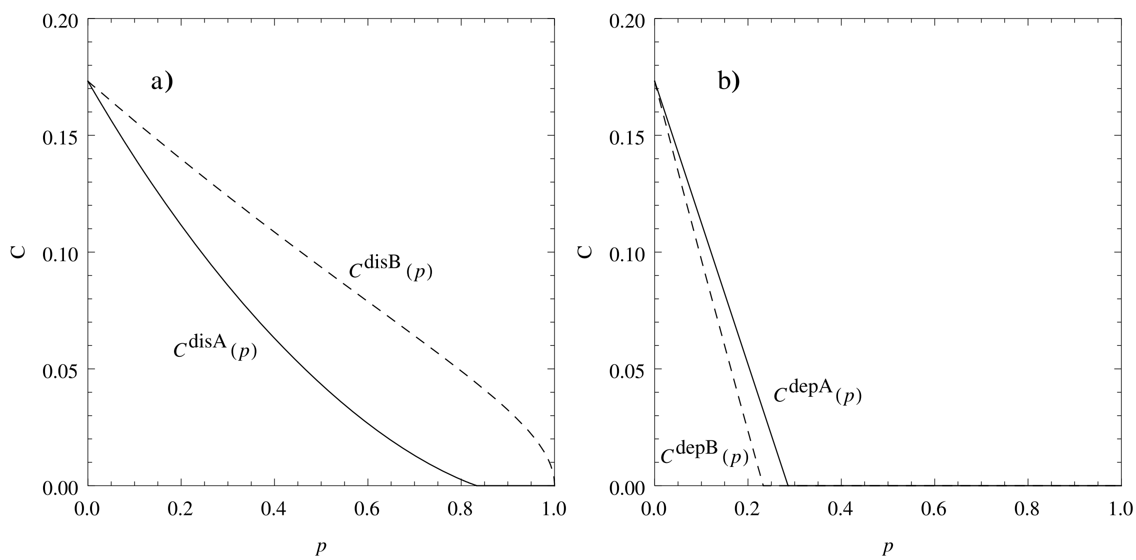

Fig. 2. Concurrence C as a measure of entanglement in the different cases of decoherence: (a) dissipation (5) of subsystem A (solid line) and subsystem B (dashed line); (b) depolarization (6) of subsystem A (solid line) and subsystem B (dashed line).

$$\rho_{AB}^{asym} = q|\psi_1\rangle\langle\psi_1| + (1-q)|\psi_2\rangle\langle\psi_2|, \tag{7}$$

where $|\psi_1\rangle = a|00\rangle + \sqrt{1-a^2}|11\rangle$ and $|\psi_2\rangle = a|10\rangle + \sqrt{1-a^2}|01\rangle$ are normalized pure state vectors.

Similarly to Ref. [1] we take the system $\rho_{AB}^0 = \rho_{AB}^{asym}|_{q=q_0,a=a_0}$ for $q_0 = 3/5$ and $a_0^2 = 3/4$. For these parameters $S_B = 0.562$, $S_{AB} = 0.673$ and $S_A = 0.688$, so such a system satisfies the inequality (1). Therefore the subsystem A can be called "quantum" and the subsystem B – "classical". The linear velocity of irreversible information flow $c_2 = 4.590 > 0$, so the causal connection has direction $A \to B$.

We consider the decoherence processes of a single subsystem from the state ρ_{AB}^0. The behavior of linear velocities of irreversible information flow $c_2^{dis\,A}(p)$ and $c_2^{dis\,B}(p)$ in the cases of subsystem A and subsystem B dissipation (5) is shown in Fig. 1. In the case of subsystem A dissipation we observe that $c_2^{dis\,A}(p)$ discontinues and changes its sign at $p = 0.444$. It means that causality changes its direction from $A \to B$ to $B \to A$. At greater p causality increases up to its maximal value: $c_2^{dis\,A}(p) \to 0$ at $p \to 0$. In the case of subsystem B dissipation we observe monotonous amplification of causal connection. Such a behavior is totally conformed with the statement that dissipation makes the corresponding subsystem into a sink of information or, in other words, into an effect.

The behavior of causality in the depolarization processes (6) is shown in Fig. 1(b). We observe a discontinuance and a change of sign of $c_2^{dep\,B}(p)$ at $p = 0.8$ in the case of the subsystem B depolarization. Similarly to the case of subsystem A dissipation, state causality of subsystem B depolarization changes its direction and then increases up to maximal value: $c_2^{dep\,B}(p) \to 0$ at $p \to 0$. In the case of subsystem A depolarization causality increases monotonically up to its maximal value. Thereby the depolarization acts in opposite way in comparison to the dissipation. Such a behavior is totally conformed with the statement that dissipation makes the corresponding subsystem into a source of information or, in other words, into a cause.

Now let us consider the behavior of an entanglement for all the four different cases of decoherence. For an entanglement measure we use the concurrence C [14]. For the initial state ρ^0_{AB} $C^0 = 0.173$.

The behavior of concurrences for the dissipation is presented in Fig. 2(a). From this figure we notice that in the case of A subsystem dissipation the entanglement decay is stronger than that one in the case of B subsystem dissipation ($C^{dis\,A}(p) < C^{dis\,B}(p)$ at $p > 0$). It means that for the dissipation process the "classical" subsystem is more tolerant in sense of entanglement persistence.

The case of depolarization is presented in Fig. 2(b). As it is evident from this figure, "quantum" subsystem is more tolerant from the standpoint of entanglement persistence in the depolarization process: $C^{dep\,A}(p) > C^{dep\,B}(p)$ at $p > 0$. So we have got the anomalous entanglement decay which was discovered in Ref. [1] for the more complicated model of depolarizing channel which involves unitary dynamics of the system.

By the comparison of Figs. 1 and 2 we can conclude that the entanglement decay is always stronger in the cases of causality reversal. Therefore *from the standpoint of entanglement persistence the decoherence acting along original causality is better than the one acting against this causality*. In other words, for entanglement persistence one should not "stroke the system against the grain". So the anomalous entanglement decay phenomenon appears in entropy maximizing processes like depolarization and does not take place in entropy minimizing processes like dissipation.

5. Conclusion

We have considered the causality behavior and entanglement decay processes in the asymmetric "quantum–classical" state (7) under two models of decoherence: the dissipation (5) and the depolarization (6). Let us list the principal conclusions.

1. The extent of system asymmetry can be characterized by the measure of causality c_2.
2. The different types of decoherence have the different impacts on causality: the subsystem under dissipation becomes an effect (information sink), the subsystem under depolarization becomes a cause (information source).
3. From the standpoint of entanglement persistence, decoherence, which is acting on asymmetric bipartite system along original causality, is better than the one acting against this causality.

Finally it should be noted that the similar conclusions are applicable not only to "quantum–classical" states, which are characterized by inequality (1), but to all the asymmetric states with unequal subsystem entropies.

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