AN APPROACH TO FINANCIAL TIME

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ABSTRACT:

Nowadays, financial managers allow Stock Market prices' evolution to be complex and there is any satisfactory and known model for the profitable prediction of Stock Market prices. There are many performance indicators, but none of them is infallible and better than the others. Moreover, it is common reading in specialised literature new methods or applications of complicated physical models to try to predict the behaviour of Stock Markets, which shows that financial models are unsatisfactory.

If the former paragraph is right, then mathematical approximation to Finance should be descriptive and qualitative, better than predictive and quantitative. Stock Market prices are neither random nor deterministic; human influence is noticeable [G. Soros (1999)], but it is not the unique. That is to say, the behaviour of Stock Markets would be chaotic [B.B. Mandelbrot (1997) and P. Ormerod (2000)]. One characteristic of chaotic systems is the fractal dimension of its graphical representation. Some models' variables have a dimension less than one, so with a Lebesgue measure of infinite in zero-dimensional space and of zero in one-dimensional space. This kind of variables "fills" some non-continuous parts of the space.

Time series of Stock Market prices are placed in a space-time of two dimensions. Price values in space and their position (order) in time. The author thinks that time series may be classified according to the dimension of its occupation of space-time:

- Deterministic: one-dimension. Time is considered space, the order is not important because knowing a price and its position you can calculate the other prices.
- Random: two-dimension. You need two dimensions, space and time (value and order), to know the dynamics of the data series. There are two sources of variability: prices (space) and its order (time). In this case, two similar prices do not involve two similar evolutions because their time positions are different.

- Fractal: dimension between one and two. Observations are put in order (time) in space (prices), but this order does not fill the time dimension because price changes are irregular and non-continuous.

Financial Time is the time of Stock Markets [H. Guitton (1970)], the speculation time, the future time (future prices are more important than present prices), jumps in prices time, ... This time conception helps to define the dynamics of Stock Market prices and the author approximates it from the estimation and description of fractal structure of time dimension of financial series.

In this paper, it is proposed to estimate the Financial Time associated to a Stock Market price series and to study the relationship between this Financial Time and the clock-time in duration and structure.

KEYWORDS: Time, dimension, fractal, Stock Markets, short-term, time series, Financial Time.

INTRODUCTION.

Stock Markets (SM), in a mathematical sense, are orderly series of pairs: prices and negotiated volume, which are limited to some rules of price formation and of accepted claims. It is common to resume these series by the daily frequency of maximum, minimum and closing prices, together with the total negotiated volume of this day. That is the fundamental information of the financial managers with the Fundamentals.

The study of financial information is possible to carry out at short or long dated. Normally, investors that want fast profits are interested in short-term analysis. In the Financial Theory it is more normal to use long-term analysis. In the present communication, the short-term analysis of financial series will be studied because the aim is to describe SM prices, which are short-term fluctuating. In this sense, the research will be about whether Time is an explanatory variable of financial series and whether the day is the right time unit.

In short-term analysis, prices are the fundamental variable because their constant fluctuation can not be represented by normal stable variables, and this constant fluctuation is the short-term dynamics of SM. That approach to short-term SM may be defended from two explanations:

1- The Efficient Market Hypothesis (EMH): prices contain all the relevant information, at least public information. Then, you can use only prices to study SM' evolution.
2- The high daily volume of SM shows that intraday operations are the commonest and then short-term operations are more important than long-term operations.

The quantitative analysis of financial series has been the commonest mathematical tools in Finance, with the aim of the forecast. This analysis includes statistics, econometric methods, Artificial Intelligence tools like Expert Systems and Neural Networks. But these mathematical and statistical methods are based in the hypothesis that SM prices' evolution is predictable, either because that evolution may be represented by a functional relation, or because the associated randomness is negligible (statistical relation). According to the estimated models, which are based in precedents tools or theories, the model's structure or the reality is stationary, that is to say, the parameter values are fix in the studied period of time. However, the fluctuating SM prices can not be represented by these models with a small error. In this sense, in the present communication, terms, ideas and tools of the Mathematical Chaos Theory are used with the aim of the SM prices' description and explanation. In this way, the author proposes to analyse the Time variable for the explanation and description of SM.

THEORY.

Time of short-term SM is the Time of the speculation, of the fluctuation. This Time is characterised by the big changes that happen in it and because shows one preference for the future of the economic agents [Guitton, H. (1970)].

In Finance, Time should not be eliminated like in Physics, where it is universalised through one numerical variable, which puts in order and allows calculate by difference the duration (Operating Time). Neither it should be handled like a biological Time, which is measured by the observed changes (inner Time). In short-term Finance, the pass of Time does not leave a trace because there is a continuous inner tendency to the change, which is in the non-realised operations. Although there exists a memory, as short-term memory as long-term memory (persistence, Monday Effect, financial Crashes). Financial Time, in the short-term vision, is one scale where the SM prices' fluctuations happens or passes. These fluctuations "contain" a tendency to the change, what shows the future perspective (expectations) of the investors and the inner uncertainty of the SM. Time is an explanatory variable of SM behaviour because Financial Time has influence in the short-term investor's decisions.

Therefore, Financial Time is one dimension of the change, where financial series may be put in order and placed (the other dimension, space dimension, it is the price) with its characteristics of persistence (short-term memory, Joseph Effect), abrupt and discontinuous fluctuations (Noah Effect), future perspective (expectations) and inner uncertainty (tendency to the change, non-equilibrium). Finally, it is a scale (different states o visions of study of the reality).

In this sense, the Financial Time is between the operating mathematical Time (numerical order and distance) and the inner biological Time (changes). In this way, the Financial Time is a regularity (scale) that allows to put in order the financial series and to measure the separation between the data. Also, that scale explains in part the fluctuation's sequence of SM prices.

The fundamental unit of analysis in Modern Financial Theory is the "Financial Capital" [Rodríguez, A. (1994)], complex binary number, if the Financial Capital is deterministic, which consists of a quantity (capital, monetary units) and the moment of time in which is placed the capital (time units). It shows that Time is important in Finance because it contains the liquidity preference of the investors and it implies that the interest rate is positive.

Thanks to the definition of Financial Time, which it is presented in this communication, financial models or reality may be classified from the Time variable:

- Deterministic: Time is a simple number, it is adimensional. It puts in order financial series, but it does not explain them. The information is contained in the space variable of the value. That order allows to represent quantities through one perfect functional relation or with negligible error. and consequently, the financial series.

- Random: Time is a variable of unitary dimension. It shows one different situation at every value. The time unit is measured by the periodicity or separation between the moments in which are the Financial Capitals, that is to say, the time interval where there is placed a unique Financial Capital and for bigger intervals there are more than one Financial Capital. In this sense, for the explanation and representation of the financial series are necessary as the space dimension (quantities) as the time dimension (moment and order of the change). The only regularity is the probability distribution, which represents the values' repetition in an time interval because it is not possible to estimate any functional relation because of the series' order. It is necessary to study individually the data.

- Fractal: Time dimension is between zero and one. Time and quantity are explanatory, but the later more than the former. The time dimension shows a regularity in the order, what it allows the generalisation of some mathematical and statistical properties of financial series, but they are not functional relations.

Deterministic time approach or the elimination of Time variable has been the traditional approach in Financial Theory through either of considering that analysis in unitary time intervals (static approximation), or of considering Time as a subscript of order, or of considering it as a numerical independent variable. The explanation is only done from monetary unit (prices). Examples are Markowitz's Theory of portfolio selection, the CAPM, the APT, econometric models, the Elliott waves, Spectral Analysis, Technical Analysis. In this approach errors are considered as negligible and, therefore, as if the approximation by means represented the reality. In this way, the deterministic or quasi-deterministic models may be interpreted as theory more than as practice, because they are easily rejected by the observation of the reality and because they are small successful in the practice. But these models are useful in financial policy to modify the reality or to try to get an economic equilibrium.

The random approach has been defended by B. Markiel (1996) and most of the present financial researchers. The idea of the SM prices follow a Random Walk is the commonest in 90's. Several estimations and experiments have been made to prove the precedent idea, but any of them have not been conclusive.

That perspective supposes to use the probabilistic interpretation in the SM analysis. In this sense, the Stochastic Calculus and the Martingale's Approach have been incorporated to Modern Financial Theory [Malliaris, A.G. (1988) and Musiela, M. (1997)] because with these probabilistic tools the financial asset's valuation and price volatility's representation are improved. Also, that approach deals with the characteristic of fat tails of financial distributions. In this case, the time unit has unitary dimension and characterises financial data, like the monetary unit. Every price is in a moment and it is not only the following of the precedent, like in a functional relation. Financial Capitals are two-dimensional and the Time has influence in the probability distributions because the parameters change as a function of time and for the data order, that does not allow to infer a functional relation among the data.

While in the deterministic approach the error is what limits the model's capacity, in the random approach the risk in the decision is the limit, because in this case the future is impredictable. It is not possible to know *a priori* the following price and the explanation of a financial series is that the observed data are one of the several possible cases, that might be happened. Therefore, that approach depends on the success in the decisions because when the success is common, then it is possible to estimate a functional or statistical relation among the data by the regularity in this success. This means that it is possible to generalise the information in a mathematical or statistical way because there is a repetitive behaviour and there is not risk in the decision.

In the middle of the precedent approaches is the fractal. This vision supposes that the time dimension is not integer, what it may be interpreted as Time characterises the financial data (the time unit is important) and as Time shows a regularity, that allows to generalise some properties of financial prices. Therefore, less fractal dimension is, nearer deterministic vision is, because the regularity “dominates” to the individuality of data.

The time dimension is between zero and one because the occupation of the dimension will be between nothing (zero-dimension) and all (one-dimension). It has not sense a dimension larger than one because in this case Time will not be able to be used like a scale of data order.

The fractal paradigm has been carried out by B.B. Mandelbrot (1997), who tries to introduce fractal dimensions, either stationary (unifractal), or variable in time (multifractal), in order to get a better representation of SM. B.B. Mandelbrot (1997) distinguishes among the Brownian motion (the standard desviation of the quantities is proportional to square root of time), the unifractals (proportional to a exponent between 0.5 and one), and multifractals (the exponent is variable in time or is time-dependent).

The Brownian motion supposes that the financial variable follow a normal distribution with mean zero and variance equal to the duration of the interval where the change happens or is measured. The unifractal supposes persistence of the time series and the fractal dimension of the time series is the inverse of the exponent, like in multifractal. The dimension is between one and two, between a line and a surface. It depends on the time dimension according to the interpretation of this communication.

E.E. Peters (1991) makes also applications to the Chaos Theory in order to explain SM and their aperiodic cycles. E.E. Peters (1991) uses the R/S methodology (unifractal) to estimate the dimension (occupation) of a financial series, which fills more than a line, but less than a surface because of its irregularity.

Other applications of unifractals and multifractals have been introduced in Econometrics like ARFIMA models, α-stable Pareto distributions, ..., but these applications suppose that the error is negligible after introducing the fractal dimension.

P. Ormerod (2000) interprets the financial data from Chaos Theory because although at aggregate level may seem a probability distribution, one analysis of the data order shows the existence of persistence, short-term memory. Normally, the data repeat the motion of the precedent.

U. Nieto de Alba (1998) considers the bifurcation's analysis, where the start of every phase is defined by a bifurcation or a multiple possibility of ways. The Time is inner to the system and an unity passes by every bifurcation. The passage of time implies a change of the parameters of the model or even in the structure's model. In each phase the model is supposed to be valid and, therefore, useful for the forecast. In Finance, that vision is not valid because the daily fluctuation often changes the prices and, as a result, the parameters. In Finance, it is not possible to fix stable periods, except at a macro level, that is to say, economic cycles. But that is a long-term analysis, what it is not the aim of the present communication.

Also, in Physics, fractal analysis and Chaos Theory has been used, by example for the estimation of a fractal space-time in order to represent the implications of Relativity Theory, which can not be contrasted in an independent four-dimension space-time [Nottale, L. (1997)]. In Physics, the analysis passes from individual level to collective or statistic level, in order to eliminate the Time. But in Finance this approximation is not valid because every moment is a different situation, in general [Frailey, F. (2000)].

Therefore, enumerated methodologies are not valid for the analysis of Time variable in Finance. Then, if you think in the classification of financial series according to the Time dimension, this approach is more explanatory than the others.

PRACTICE.

Classical applications of Chaos Theory is not successful in Finance, except when the aim is the description and not the forecast. In this sense, used mathematics are simple because the author pretends to explain and describe SM prices and not to represent and forecast them. As SM are intuitively understandable, then it should be possible to elaborate a model that describes this reality. There are more things than probability. In a short-term analysis the important variables are two: prices and Time. Other variables are not considered.

In the communication three financial series are analysed. These series represent, in the opinion of the author, the commonest cases in SM. The data are the difference between closing price and the maximum price of next day. That represents a possible rational short-term behaviour of a stereotyped investor. He buys a day and he sells the next day. By restrictions of periodicity of data it is not possible to consider other short-term strategies. The model is based in a daily profitability rate by the data availability, but the author considers that it can represent short-term SM.

The three prices series are Telefónica (TEL), between the years 1986 and 1999, the General Index of Madrid SM (IGM), between the years 1986 and 1999, and Amper (AMP), between 1987 and 1999. The last series is more fluctuating than the other two.

With a classic econometric analysis (ARMA), the outcomes are very bad and the R^2 regression coefficient is smaller than 0.05, except in AMP case that it arrives at 0.28. This means that it exist a small short-term memory. But if data are analysed, for the TEL case the 80.2% of data are positive rates, for IGM case 62.4% and for AMP case 80.8%. That is to say, there exist persistence in the three series and then there is a sign of chaos. Moreover, in the majority of cases the following rate has the same sign of the precedent, what is the definition of persistence (short-term memory in order).

The autocovariances are near zero in the three cases like its autocorrelation coefficient, what shows a lack of functional and statistical relation among the prices, principal variable in short-term analysis, according to its order.

Other classic methods have also bad outcomes, for example, Spectral Analysis (no significant cycle can be estimated), Elliott waves (there are no observed alternative changes in short-term analysis), Stochastic Calculus (the distribution of prices does not seem a normal distribution with stable or increasing variance; irregular fat tails).

Also, the regression of data about the Time variable has a small R^2. However, if one works with accumulated data the R^2 increases to 0.99 about the Time and about the precedent accumulated rate. But this “good” statistical fitting is fictitious because the mean error in the first case is between 4% and 7% and the mean profitability rate is between 5% and 8%, according to each series. Therefore, the error is about the 80% for the investor’s interest. In the second case, the absolute mean error is similar to the mean profitability rate.

However, if a data division is made in three categories: negative rates, rates between zero and 2%, and rates higher than 2%, then the outcomes improve. With this separation of data and working with these categories like dummy variables, the statistical fitting about the normal rates increases from 0 - 0.05, in the regression about a constant and about the Time, to 0.12 - 0.65, in the regression about the three dummy variables. For TEL and IGM cases the statistical fitting increases to 0.65, and in AMP case to 0.12. In the cases of TEL and IGM, the error is reduced about 20% in relation with the fitting of accumulated rates about the Time. In AMP the outcomes are not so good.

This means that with the three categories a Financial Time is introducing in short-term analysis. This Time is in one part an inner Time, because it depends on the changes' value, and in the other part an operating Time, because the categories are numerical and the possible statistical dependence between data and Time is also numerical.

This Financial Time is similar to the classification of time series that the author proposes in the communication. That is to say, if it is possible a perfect or good statistical fitting about the Time, then it is the deterministic case because the Time is adimensional (Time puts in order only the series) and its units are the same than prices (monetary units). If it is not possible a good statistical fitting about some limited number of Time categories (separation of moments according to the changes' value; regularity), then the Time variable has unitary dimension because it is not possible to fit a regularity and every moment shows a different situation. If it is possible a good statistical fitting about some limited number of Time categories, then the Financial Time is fractal, because in part it represents the prices (regularity) and in other part it characterises the data (individuality).

The good statistical fitting is robust because if the financial series are divided in different periods with a duration higher than two years, the R^2 of the regression of rates about the three categories are similar. That may be interpreted as investors take decisions in a different way if there are loss, if there is a small profitability rate or if there are high profitability rates. The division between normal and high profitability rates is 2%, it may be explained through because in Spain the commissions by operating in SM are between 1% and 2%. Then a net gain is guaranteed with a profitability rate higher than 2%.

For TEL and IGM cases are possible to affirm that the time dimension is fractal, but for AMP case is not clear. A fractal Financial Time shows some degree of data dependence about Time and some individuality of data.

The degree of data dependence about the Time may be approximated by the improvement of the error in relation with the mean of absolute value of the difference between the error and the mean of the rates. The last mean is a measure of the error without any dependence. The outcomes are:

TEL case, an improvement of 12.5%.
IGM case, an improvement of 15.6%.
AMP case, there is no improvement.

That means that for TEL and IGM cases there exist some data dependence on Time and in the AMP case any. Then AMP series is random and the other two are fractal.

In order to estimate the fractality of Financial Time or individuality of data, that may be approximated by the irregularity in the changes of defined categories. The TEL series has, in a 45.84% of cases, a change of category between two following data. As the proportions of categories in this series are 19.77% losses, 16.30% more than 2%, and 63.93% between 0 and 2%. Then the maximum probability of the precedent irregularity is (independence among categories): $1 - (0.1977^2 + 0.163^2 + 0.6393^2) = 0.5256$. Therefore, the proportion of irregularity of TEL series about the maximum is the 87.20%. This means that the series is near randomness (unitary case).

For the case of IGM, AMP case is random because does not show dependence on Time, the irregularity is the 41.07% and it represents a 77.79% about the maximum. Then it is logic that the degree of dependence on Time of IGM series is higher than TEL case, because its irregularity is smaller.

The last proposal of this communication was the study of the work time unit of investors. In the three cases, in despite of its different time dimension, the relevant time unit is the day. This fact may be explained for the high irregularity or fractality of three series in the Time variable. Possibly, if the initial periodicity of data was less than the day, the results would be different.

The last outcome is deduced because when one analyses the proportion of operation with loss at different time scales, that is to say, when the separation between the buy claim and sell claim is of a day, two days, ... In this analysis, it may be observed that the smallest proportion is for a separation of one day. For higher duration the proportion of loss increases as you can corroborate in the next table:

Cases of LOSS according to the interval of buying and selling

DAYS	**TEL**	**IGM**	**AMP**
1	674	1252	628
2	1041	1418	1071
3	1142	1417	1204
4	1214	1406	1278
5	1221	1425	1331
6	1224	1439	1361
7	1240	1433	1402
8	1258	1413	1432
9	1264	1415	1445
10	1271	1418	1474
1 MONTH	1245	1321	1558
1 YEAR	811	881	1676

Therefore, a rational investor will take the decisions of selling a day after the decision of buying, because he will have the smallest probability of loss and moreover he will have the highest accumulated profitability.

CONCLUSION.

The presented study can not be generalised to a theory about the analysis of Financial Time, but at least the author thinks that the intuitive approximation which has been presented is a first stage in the study of Financial Time. The simple methodology of classification of financial series has a significant explanatory character and it is valid for the study of Financial series. Then the author thinks that the communication has some quality of research.

The dynamics of short-term SM is complex, but that does not imply that it can not explain the SM in a simple way, like through the classification of financial series according to the associated time dimension. In this sense, there are deterministic time series (functional or statistical relation), random time series (not relation, individual data) and fractal time series (in the middle). Then, depending on the time dimension the relevant unity of Time can vary. In the random case is the periodicity of data, in the deterministic case is the total interval (adimensional Time) and in the fractal case, it depends on the irregularity of data. If this irregularity is high, then the time unit is near periodicity of data and more regular time series is, higher time unit is.

Finally, the made practical analysis shows the characteristics of Financial Time, which are been described in the theoretical part. This means, Financial Time in short-term analysis shows persistence and short-term memory (high repetition of motion sign between a rate and the following rate), abrupt and discontinuous fluctuations (high irregularity, inner Time), future perspective (selling if there is gain), inner uncertainty (non-equilibrium), numerical dependence on Time (operating Time), and it is a scale (regular categories of classification, between financial series and in rescaled data or with different periodicity).

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OTHER INFORMATION.

TELEFÓNICA (TEL):
Data number: 3412.
Daily rates TEL = -0’012·Ip + 0’033·Ig + 0’007·Io; R^2 = 0’6526.
Mean error of regression: 0’0099.
Mean of absolute value of the difference between the error and the mean of rates: 0’0113.
Improvement of regression in relation with the mean: 12’47%.
Composition data: 19’77% loss (Ip), 16’30% gain higher than 2% (Ig), and 63’93% other (Io).
45’84% of data irregularity (alternation of different time categories).

GENERAL INDEX OF MADRID SM (IGM):
Data number: 3352.
Daily rates IGM = -0’0076·Ip + 0’028·Ig + 0’0068·Io; R^2 = 0’6461.
Mean error of regression: 0’0067.
Mean of absolute value of the difference between the error and the mean of rates: 0’0080.
Improvement of regression in relation with the mean: 15’62%.
Composition data: 37’36% loss (Ip), 5’22% gain higher than 2% (Ig), and 57’42% other (Io).

41’07% of data irregularity (alternation of different time categories).

AMPER (AMP):
Data number: 3272.
%VAR. AMP = -0’031·Ip + 0’064·Ig + 0’0079·Io; R^2 = 0’1164.
Mean error of regression: 0’095.
Mean of absolute value of the difference between the error and the mean of rates: 0’030.
Improvement of regression in relation with the mean: ANY.
Composition data: 19’20% loss (Ip), 32’74% gain higher than 2% (Ig), and 48’06% other (Io).
50’58% of data irregularity (alternation of different time categories).