

Superfluid Mass-Energy Densities of Nonlocal Particle and Gravitational Field

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Abstract Dynamical gravitational and geodesic equations are derived for superfluid densities of nonlocal self-coherent particles. The geometrized gravitational particle is the r^{-4} distribution of inertial mass that balances Ricci curvatures in the Einstein equation without the right-hand side. The spatial energy integral of such an infinite radial particle is finite and determines its nonlocal gravimechanical charge for energy-to-energy interactions with other nonlocal particles. Non-empty space of the flat material world is filled continuously by overlapping energy-flows of all nonlocal particles and their fields.

Keywords Nonlocal particles · Self-coherent energy distributions · Spatial overlap of matter · Superfluid densities in potential fields

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Quantum physics maintains that a distributed density $n(x)$ of one interaction-free electron obeys the same Sommerfeld quantization rule as the nonlocal self-organization of two superelectrons within the distributed Cooper boson. One may say that nonlocal material densities of all free particles equally exhibit superfluid states along any Feynman's paths $\int P_\mu dx^\mu$ with synchronized particle's time, $d\tau(x) \equiv g_{0\mu} dx^\mu / \sqrt{g_{00}} = 0$. However, a closed line contour in 3D volume with a single-valued potential $\chi(x)$ for the canonical

four-momentum density $n(x)P_\mu(x) = -n(x)\nabla_\mu\chi(x)$ may keep the relativistic flux quantization only under strict spatial flatness [1]. In other words, Feynman's self-coherent distributions or nonlocal self-organizations of elementary matter through superfluid potential states are impossible in principle in the widely accepted interpretation of Einstein's relativistic physics [2, 3] under curved 3D spaces. No matter how slightly gravitation might curve 3D laboratory space, strict quantization and self-coherent superconductivity of material carriers would never be expected in such an imaginary world.

Contrary to delocalized particles in quantum mechanics and condensed matter physics, the gravitational particle is still considered by specialized General Relativity (GR) journals as a localized energy, represented exclusively by the right-hand side of the 1915 Einstein equations $(R_\nu^\mu/\kappa) - \delta_\nu^\mu(R/2\kappa) = T_\nu^\mu$, with $\kappa/c \equiv 8\pi G/c^3 \equiv 1/\Upsilon$. This pre-quantum approach to the (distributed) energy-source seems not only obsolete in the 21st century, but also suggests unrealistic, curved 3D spaces around a point electron (without its Gauss flux conservation over any closed surface, for example). Schwarzschild metric solutions to the Einstein equations at the empty-space paradigm have no mathematical errors, but leave no room for Sommerfeld quantization of charged distributed particles, as well as for acceptable quantization of their gravitational fields.

At the same time, flat 3D space in the 4D interval $ds^2 = [1 + (Gm/c^2r)]^{-2}c^2 dt^2 - \delta_{ij} dx^i dx^j$ may be self-consistently suggested by the static Einstein equation $[R_0^0 - (R/2)]/\kappa = 0$, when one associates the nonlocal radial distribution of elementary matter with the Ricci curvatures $R_{\mu\nu}$ and R rather than with the artificial point-energy term $T_{\mu\nu}$ [4]. The integration of particles into spatial structures of their fields was assumed by Einstein: 'We could regard matter as being made up of regions of space in which

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the field is extremely intense. . . . There would be no room in this new physics for both field and matter, for the field would be the only reality,' translation [5]. Such a condensed matter directive of the GR author advises that the point mass action $S = - \int mc ds$ could be extended in some way on mass densities of active m_a (gravitational field) and passive m_p (inertial, mechanical particle) masses, with $m_a = m_p$ due to the Principle of Equivalence.

We relate active and passive mass densities of the superfluid carrier of energy to the scalar Ricci curvature, $(m_a + m_p)n(x) = R(x)/\kappa \equiv g^{\mu\nu}(x)\gamma c^{-1}R_{\mu\nu}(x)$. This scalar curvature or scalar mass density of a self-coherent organization of elementary paired energies takes place in all points of its material 4D manifold, described (by a local observer) through the specific metric tensor $g_{\mu\nu}$ and the specific 4-volume element $\sqrt{-g}dx^4 = (ds/dx^0)dx^1dx^2dx^3dx^0$ for every elementary space–time–energy self-organization. Spatial overlap of a selected nonlocal particle-field with all other nonlocal elements of the undivided material world generates local external fields in the metric tensor $g_{\mu\nu}$ and curved 4D interval ds , both introduced for the local mass density of one selected nonlocal particle.

We shall not consider reality-independent options of Riemann–Finsler geometry and, therefore, accept only those metrics $ds = \sqrt{d\tau^2 - dl^2}$ for elementary space–time–energy self-organizations, where six (from ten “independent”) components of the GR metric tensor $g_{\mu\nu}$ are bound by six inherent symmetries, $\gamma_{ij} \equiv (g_{oi}g_{oj}/g_{oo}) - g_{ij} = \delta_{ij}$, with $g_{\mu\nu}g^{\nu\rho} = \delta_\mu^\rho$ and $g_{\mu\nu}\delta g^{\nu\rho} = -g^{\nu\rho}\delta g_{\mu\nu}$. These physical symmetries were derived from the coherent tetrad generalization of the Special Relativity (SR) interval [4]. They keep the Euclidean 3D sub-interval, $dl^2 \equiv \gamma_{ij}dx^i dx^j = \delta_{ij}dx^i dx^j$, in all covariant equations for distributed densities of nonlocal elementary matter. Due to the six physical bounds for the pseudo-Riemannian metric tensor $g_{\mu\nu}$, we can expect only four gravitational field equations from the action variations with respect to four independent metric components, $g_{o\mu}$, which are responsible for anisotropically dilated (Finslerian) time $d\tau(x, dx)$ of a moving particle. Another four geodesic equations for self-coherent densities of the nonlocal elementary carrier come from the Lagrange variations over the material density coordinates x^μ :

$$\begin{aligned}
 0 = \delta S &= -\delta \int_{s_1}^{s_2} (m_a + m_p)c ds = -\delta \iint d^3x \gamma R(x)u_\mu dx^\mu = -\delta \int_{s_1}^{s_2} \int d^3x g^{\rho\lambda} \gamma R_{\rho\lambda}(x) ds \\
 \left\{ \begin{aligned}
 &= - \int \int d^3x \{ \gamma Ru_\mu d\delta x^\mu + dx^\nu \partial_\mu (\gamma Ru_\nu) \delta x^\mu \} = \int \dot{s} d^4x \{ u^\nu \nabla_\nu (\gamma Ru_\mu) - u^\nu \nabla_\mu (\gamma Ru_\nu) \} \delta x^\mu \\
 &= - \int_{s_1}^{s_2} \int d^3x (ds \gamma R_{\rho\lambda} \delta g^{\rho\lambda} + \gamma R \delta ds + ds g^{\rho\lambda} \delta R_{\rho\lambda} \gamma) = \int \dot{s} d^4x \gamma (2R_{\rho\lambda} g^{\mu\rho} g^{\nu\lambda} - Ru^\mu u^\nu) g_{o\nu} \delta g_{o\mu}.
 \end{aligned} \right. \tag{1}
 \end{aligned}$$

Here the omitted term $\delta(R_{\rho\lambda}\gamma)$ was replaced at first with the vanishing integration over the 3D ‘surface’ in infinity due to the Gauss divergence theorem, applied to the curved 4D with the volume determinant $\sqrt{-g} = ds/dx^0 \equiv \dot{s}$. The 1915 action variations of the Ricci curvature correspond to Hilbert 4D spaces with a non-moving gravitational source (regarding a local observer) that resulted in $\sqrt{-g} = ds/dx^0 = \sqrt{g_{oo}}$. Such rest-frame material spaces may justify only the static source equation $R_o^o = R/2$ or only post-Newtonian gravi-‘electric’ fields in the 1915 Einstein equations (which is not quite suitable for three gravi-‘magnetic’ fields of a moving source).

Finally, the Einstein directive toward nonlocal continuous sources resulted in four-vector geodesic equations (2a) for superfluid mass-densities,

$$\begin{cases}
 u^\nu \nabla_\nu (\gamma Ru_\mu) = u^\nu \nabla_\mu (\gamma Ru_\nu) \\
 \Rightarrow d(Ru_\mu)/ds = \partial_\mu R, & \tag{2a} \\
 I_o^\mu \equiv \gamma (2R_o^\mu - Ru_o u^\mu) = 0, & \tag{2b}
 \end{cases}$$

as well as in dynamical gravitational equations (2b) for four balanced tensor densities I_o^μ , associated with the energy-flow density $(\gamma Ru_o)u^\mu$ of superfluid active and passive mass-currents in their joint four-vector γRu^μ .

Notice that the Ricci curvature $\gamma R_o^\mu = \gamma g^{\mu\nu} R_{\nu o}$ of the elementary space–time–energy self-organization is balanced by the tensor energy-flow $(nm u_o)u^\mu$, rather than by the GR mass-current four-vector $nm u^\mu$. This means that variable mechanical energies, not scalar mass invariants, are ultimate charges in the quantitative Machian relativism (2), as was qualitatively predicted for nonlocal gravitation and inertia [6]. A global dynamical balance, $\sum_1^\infty I_o^\mu(x) = 0$, for the local summary of all gravimechanical energy densities is valid for the total spatial overlap of all continuous particles and their fields.

One can formally reiterate the 1913 Einstein–Grossmann geodesic equation, $mu^\mu \nabla_\mu u_\nu \equiv m Du_\nu/ds = 0$, from the point mass approximation of nonlocal matter in (2a) by replacing the probe particle density with the delta-operator density of the constant point mass, $\gamma R(x) \rightarrow 2m\delta(x)$. However, the similar delta-operator approximation of mat-

ter for the ‘point’ energy-source would assign different space arguments to the field and the particle in the continuous source equation (2b). This approximation contradicts in principle not only to our non-empty space physics, but also to the mathematical grounds for differential equations with analytical functions.

Can the empty space paradigm submit unbeatable reasons to withstand analytical mathematics of non-empty space alternatives and nonlocal physics of self-coherent superfluid matter? In order to answer this basic question, we rely on the original GR formalism [2, 3] in our condensed matter approach to superfluid distributions of nonlocal particles with mutual gravitation and inertia. Let us consider that a distributed passive-inertial mass m_p of a probe radial particle is centered around one point in a static central field of a gravitational source with the motionless active mass M_a and active energy E_a . Then the GR passive-inertial energy E_p of the moving probe particle,

$$E_p \equiv \frac{m_p c^2 \sqrt{g_{oo}}}{\sqrt{1 - v^2 c^{-2}}} \equiv K + U_o, \tag{3}$$

may be associated with the SR mechanical energy K and the GR gravitational energy U_o . There is no need at this moment and later to employ Newtonian references from the mass-to-mass gravitation for the potential energy, which is so far an unknown function of radial distances between centers of spherical symmetries of interacting partners, $U_o = U_o(r)$. One can use instead SR energy references, $K \equiv m_p c^2 / \sqrt{1 - v^2 c^{-2}}$, in order to define $U_o(r)$. Then nonlocal metric gravitation for superfluid densities (2) may be discussed as the self-contained SR-GR theory for energy-driven interactions.

Based on ‘new’ SR references for the mechanical energy K in (3), one can represent the GR metric component g_{oo} of the pseudo-Riemannian metric tensor $g_{\mu\nu}$ in the following way:

$$\begin{aligned} \sqrt{g_{oo}} &\equiv \frac{K \sqrt{1 - v^2 c^{-2}}}{m_p c^2} + \frac{U_o \sqrt{g_{oo}}}{E_p} \\ &\equiv \frac{K \sqrt{1 - v^2 c^{-2}}}{m_p c^2 (1 - U_o E_p^{-1})} \equiv \frac{1}{1 - U_o E_p^{-1}}. \end{aligned} \tag{4}$$

This function determines the physical time rate, $d\tau = \sqrt{g_{oo}} dt$, of a motionless local observer, for whom the considered gravitational source is static and $g_{oi} = 0$ in $d\tau^2$. The active scalar mass M_a possesses a distributed GR energy $E_a = M_a c^2$ in the rest-frame of references. A spatial density of this active (source, field) energy contributes together with the passive-inertial (sink, particle) energy density to the zero balanced energy-flows, $I_o^\mu = 0$, for two bound elementary distributions (geometrized gravitational field aside with geometrized inertial particle). The 1907 Principle of Equivalence for heavy and inertial masses should be discussed in

such a non-empty space paradigm as a strict balance of active and passive energy fractions in every continuous carrier of gesamt (=whole) energy.

The Ricci tensor component R_o^o for a selected elementary couple of inseparably paired (active–passive, source–sink, ying–yang) energies is contributed both by the active density of the distributed source-energy E_a and by the equal passive-inertial energy density of the same non-local carrier. Two non-vanishing affine connections, $\Gamma_{io}^o = \partial_i g_{oo} / 2g_{oo}$, $\Gamma_{oo}^i = \partial_i g_{oo} / 2$, with the post-Newtonian potential $c^2 W \equiv -c^2 \ln(1/\sqrt{g_{oo}})$ can be traced for the Ricci tensor formalism in Cartesian coordinates for rest-frame static options of the dynamical equation (2b):

$$\begin{aligned} \frac{R}{2} = R_o^o &= g^{oo} R_{oo} = g^{oo} \partial_i \Gamma_{oo}^i - g^{oo} \Gamma_{oo}^i \Gamma_{io}^o \\ &= \nabla^2 W + (\nabla W)^2. \end{aligned} \tag{5}$$

The static local density of the elementary radial source assumes $g_{oi} = g^{oi} = 0$, $g^{oo} = 1/g_{oo}$, and $g_{\alpha\beta} = -\delta_{\alpha\beta}$ for flat-space gravitation. Many relativists tend to drop the quadratic (particle) term $(\nabla W)^2$ next to the ‘linear’ (field) term $\nabla^2 W$ for the weak ($-W \approx -U_o/E_p = +\text{const}/r \ll 1$) field reading of the Ricci–Tolman mass-energy R_o^o in the static Einstein–Poisson equation. Such an erroneous approach to the Ricci tensor formalism contradicts to the Principle of Equivalence which requests the universal local identity, $\nabla^2 W \equiv (\nabla W)^2$, of active and passive mass-energies as for strong fields, as well as for weak fields. Even without references on Einstein’s physics for geometrized continuous particles, it is not reasonable mathematics when one claims for the weak field limit that $\nabla^2 W \equiv r^{-1} \partial_r^2 (rW) \approx -r^{-1} \partial_r^2 \text{const} \equiv 0$ is the largest field term in the analytical structure of R_o^o . In fact, there is no need to double matter by the right-hand side of the Einstein equation $\Upsilon R_o^o - (\Upsilon R/2) = T_o^o$ once the quadratic field term, $\Upsilon (\nabla W)^2$, has already counted the continuous particle through the standard Ricci curvatures.

Contrary to the empty space interpretation of gravitation with non-analytical (operator) point particles, non-empty space physics explains Einstein’s local equivalence of active and passive mass-energy densities quantitatively for every radial carrier of gesamt energy,

$$\begin{aligned} \mu_a c^2 &\equiv \frac{c^2 \nabla^2 W c^2}{4\pi G} \equiv -\frac{c^4}{4\pi G r^2} \partial_r \left(r^2 \partial_r \ln \frac{1}{\sqrt{g_{oo}}} \right) \\ &= \frac{c^4}{4\pi G} \left(\partial_r \ln \frac{1}{\sqrt{g_{oo}}} \right)^2 \equiv \frac{c^4 (\nabla W)^2}{4\pi G} \equiv \mu_p c^2. \end{aligned} \tag{6}$$

Peculiarity-free solution $1 - U_o E_p^{-1} \equiv 1/\sqrt{g_{oo}} = C_1 r^{-1} + C_2$ of the nonlinear Poisson equation (4) for vector field $\mathbf{w} \equiv -\nabla W$ depends on two constants, C_1 and C_2 . One constant can be defined ($C_2 = 1$) due to the SR asymptotic behavior

of the GR metric, where $g_{oo}(\infty) \rightarrow 1$. The other constant ($C_1 = GM_a/c^2$) can be found after the volume integration of the active mass-energy density in the static equation (6):

$$\begin{aligned} M_a c^2 &= \int_0^\infty \mu_a(r) c^2 4\pi r^2 dr \\ &= -\frac{c^4 r^2}{G} \partial_r \ln(1/\sqrt{g_{oo}}) \Big|_{r \rightarrow 0}^{r \rightarrow \infty} \\ &= \frac{c^4 r^2 \partial_r (U_o E_p^{-1})}{G(1 - U_o E_p^{-1})} \Big|_{r \rightarrow 0}^{r \rightarrow \infty}. \end{aligned} \quad (7)$$

The radial potential $U_o E_p^{-1} = -C_1 r^{-1}$ of the active mass-energy charge $E_a = M_a c^2 = c^4 C_1 / G$ for the probe (passive-inertial) gravitational charge E_p corresponds to the energy-to-energy attraction law,

$$U_o(r) = -E_p \frac{G E_a}{c^4 r}, \quad (8)$$

in self-contained GR (with only SR energy references).

Again, we specified two constants, $C_1 = GM_a/c^2 \equiv r_o$ and $C_2 = 1$, in (4) and (5) through the local Principle of Equivalence (6) and the SR asymptotic references. Therefore, we specified the GR metric tensor component $g_{oo}(r) = (1 + r_o r^{-1})^{-2}$ and the invariant scalar density of the nonlocal radial particle, $n(r) = R\Upsilon/(M_a + M_p)c = r_o/4\pi r^2(r + r_o)^2$, instead of the Dirac operator $\delta(r)$, without references on Newtonian mass-to-mass gravitation. Recall that the gravimechanical energy E_p can be considered as a constant passive-inertial charge only in constant external fields, when $\partial_o g_{\mu\nu} = 0$. Relocations of ‘distant’ radial bodies change the probe energy-charge E_p or inertia in full agreement with Mach’s ideas [6], which are inseparably embedded into the self-contained SR-GR theory for energy-to-energy gravitation of nonlocal bodies.

Now one can apply the local Principle of Equivalence (6) for one static carrier to the static (for simplicity) world overlap of all active (gravitational) and passive-inertial (mechanical) continuous mass densities

$$\frac{c^2 \Delta \hat{W}(\mathbf{x})}{4\pi G} \equiv \frac{c^2 [\nabla \hat{W}(\mathbf{x})]^2}{4\pi G}, \quad (9)$$

in any point of consideration \mathbf{x} of the non-empty Universe. Here the post-Newtonian world potential, $c^2 \hat{W}(\mathbf{x}) \equiv -c^2 \ln[1/\sqrt{\hat{g}_{oo}(\mathbf{x})}]$, from all energy carriers k is defined by the static metric component $\hat{g}_{oo}(\mathbf{x}) = [1 + \sum_{k=1}^\infty r_{ok}/|\mathbf{x} - \mathbf{X}_k|]^{-2}$. One should use in the nonlinear equality (9) the strict analytical Laplacian $\Delta \sum_{k=1}^\infty r_{ok}/|\mathbf{x} - \mathbf{X}_k| \equiv 0$ for the sum of r^{-1} radial potentials, rather than the modern operator innovation $\Delta \sum_{k=1}^\infty r_{ok}/|\mathbf{x} - \mathbf{X}_k| \equiv -4\pi \sum_{k=1}^\infty r_{ok} \delta_k(\mathbf{x} - \mathbf{X}_k)$ for formal introductions of non-physical point particles with localized passive-inertial masses, $M_{pk} \equiv M_{ak} = r_{ok} c^2 / G$.

The static GR three-force, for example [7], depends only on one component of the local metric tensor $g_{\mu\nu}$,

$$\begin{aligned} \mathbf{f} &\equiv \frac{m_p}{\sqrt{1 - v^2 c^{-2}}} \nabla \ln \frac{1}{\sqrt{\hat{g}_{oo}}} \equiv \frac{m_p \sqrt{\hat{g}_{oo}}}{\sqrt{1 - v^2 c^{-2}}} \nabla \frac{1}{\sqrt{\hat{g}_{oo}}} \\ &= -E_p \sum_{k=1}^\infty \frac{G E_{ak}(\mathbf{x} - \mathbf{X}_k)}{c^4 |\mathbf{x} - \mathbf{X}_k|^3}. \end{aligned} \quad (10)$$

This force is exerted upon the probe passive energy $E_p \neq m_p c^2$, which is the only measure of body’s inertia or its gravimechanical charge. The strong field intensity \mathbf{f}/E_p corresponds in (10) to the Inverse Squared Law and the linear superposition of observable attraction forces in non-empty material space of overlapping self-coherent particles with the Bohr–Sommerfeld quantization. This flat space keeps the Gaussian surface flux from centers \mathbf{X}_k of radial masses $M_{ak} n_k$. Notice that the strong field equations (9)–(10) can be analytically satisfied (without approximations) due to the logarithmic properties of the multi-particle gravitational potential $\hat{W}(\mathbf{x})$.

It is worth noting once more that the dynamical gravitational equation $\sum_{k=1}^\infty J_o^\mu(x) = 0$ for all overlapping elementary energy carriers (or superfluid radial particles paired locally to their radial fields) assumes that every mechanical carrier continuously occupies the entire Universe despite the ultrashort gravitational scales $r_o = Gm/c^2$ of elementary radial matter. Indeed, scalar Ricci invariants of the passive and active mass-densities (of the radial carrier with equal passive/active scalars $m_p = m_a = m$),

$$\begin{aligned} \mu_p(r) \equiv \mu_a(r) &= m \frac{r_o}{4\pi r^2 (r_o + r)^2} \\ &= \frac{c^2}{4\pi G r^2} \frac{1}{[1 + (rc^2/Gm)]^2}, \end{aligned} \quad (11)$$

exist everywhere, at all radial distances in the nonlocal r^{-4} microcosm for each self-coherent particle-field formation. Electrically bound elementary carriers of active and passive radial GR energies constitute nonlocal molecules, nonlocal mechanical bodies, nonlocal planets, etc. Ultrashort transition scales, $r_o = Gm_{\text{atom}}/c^2 \approx 10^{-54} - 10^{-52} m$, for bodies’ visual boundaries are far beyond the top limit $10^{-18} m$ for modern measurements of space (and far beyond the human perception level). Nonetheless, all bulk and surface atoms of visual macroscopic bodies are nonlocal astro-distributions of radial active and passive masses. Continuous fields and continuous particles are (yin–yang) paired energy entities in nonlocal energy-to-energy gravitation. Quantum mechanics can extend the classical surface sharpness r_o up to the Planck length (while thermodynamics can extent averaged surfaces even to higher thicknesses), but cannot disregard the invisible nonlocality of the r^{-4} classical bodies. Fast

mutual energy exchanges within ensembles of radial particles provide time-averaged reconfigurations of self-coherent energy states under laboratory observations. Therefore, they might not reliably justify in the near future the instantaneous superfluid overlap of nonlocal elementary masses with mutual spatial penetrations.

Nonetheless, non-empty material space with ‘absurd’ Newtonian ether, specified by our astro-distribution (11) with the analytical elementary density $n(r) = r_o/4\pi r^2(r + r_o)^2$ instead of the delta-operator density, differs in principle from Schwarzschild’s ‘point source–empty space’ modeling of physical reality. And the Birkhoff theorem for empty (but curved) 3D spaces cannot be relevant to metric solutions for the nullified tensor curvature, $2R_o^\mu - Ru_o u^\mu = 0$, for joint geometrization of the nonlocal superfluid particle and its field in their non-empty (but flat) space. By closing, the flat-space dynamics (2) of nonlocal mass-energy carriers and the energy-to-energy attraction (8) between Machian (variable) passive and active charges can be used to criticize ad hoc

dogmas of the empty space paradigm with the obsolete point mass-energy peculiarities.

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